Comment on BLT sumrule

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Spin sum rules

There are two spin sum rules that relate integrals over distribution functions to (local) QCD operators. One for *longitudinal spin*,

$$\begin{aligned}\frac{1}{2} &= \frac{1}{2}\sum_q \int_{-1}^{1} dx \Delta q(x) + \int_0^1 dx\, \Delta G(x) + L^q + L^G \\ &= \frac{1}{2}\sum_q \int_0^1 dx\, (\Delta q(x) + \Delta \overline{q}(x)) + \int_0^1 dx\, \Delta G(x) + L^q + L^G, \qquad (1)\end{aligned}$$

and one for transverse spin (O. Teryaev, B. Pire and J. Soffer, hep-ph/9806502; P.G. Ratcliffe, hep-ph/9811348)

$$\begin{aligned}\frac{1}{2} &= \frac{1}{2}\sum_q \int_{-1}^{1} dx g_T^q(x) + \int_0^1 dx\, \Delta G_T(x) + L_T^q + L_T^G \\ &= \frac{1}{2}\sum_q \int_0^1 dx\, \left(g_T^q(x) + g_T^{\bar{q}}(x)\right) + \int_0^1 dx\, \Delta G_T(x) + L_T^q + L_T^G \qquad (2)\end{aligned}$$

In fact these two sum rules are expected to have the same contributions, at least for the quark spin part, where the equality is just the Burkhard-Cottingham sumrule. The equalities for the various terms are a consequence of Lorentz invariance. At the operator level the transverse spin sum rule involves quark-quark-gluon operators, exactly what one would expect since partons correspond to the quanta of good quark and gluon fields in front form quantization and J_T^i is not a good operator.

Tensor charge

There is a sumrule for transverse spin polarization. It relates the integral over $h_1^q(x) = \delta q(x) = \Delta_T q(x)$ to the tensor charge.

$$\sum_q \int_{-1}^{1} dx\, \delta q(x) = \sum_q \int_0^1 dx\, \left(\delta q(x) - \delta \overline{q}(x)\right) = g_T. \qquad (3)$$

The local operator involved is $\overline{\psi}(0)\, \sigma^{\mu\nu}\gamma_5\, \psi(0)$ (see Elliot's notes).

Interpretation as spin densities

The leading twist distribution functions $f_1^q(x) = q(x)$, $g_1^q(x) = \Delta q(x)$ and $h_1^q(x) = \delta q(x)$ can be interpreted as spin densities. They are 'quadratic' operators for good fields,

$\psi_+(x) \equiv P_+\psi(x) = \frac{1}{2}\gamma^+\gamma^-\psi(x)$ after taking (spin) projections, $P_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$ and $P_{\uparrow/\downarrow} = \frac{1}{2}(1 \pm \gamma^1\gamma_5)$. These spin projectors commute with P_+. One has

$$q(x) = q_R(x) + q_L(x) = q_\uparrow(x) + q_\downarrow(x), \tag{4}$$

$$\Delta q(x) = q_R(x) - q_L(x) \tag{5}$$

$$\delta q(x) = q_\uparrow(x) - q_\downarrow(x), \tag{6}$$

These are certainly prime quantities that one wants to know in our quest of understanding the structure (including polarization) of the nucleon.

Transverse spin

(a) From the interpretation one expects

$$\sum_q \int_0^1 dx\ (\delta q(x) + \delta \overline{q}(x)) \tag{7}$$

to have a meaning as 'transverse spin'. It certainly is a measure for transverse polarization of quarks and antiquarks, but there is no local operator to which it can be equated.

One can write down an operator expression, but it is nonlocal. The starting point is $\epsilon(x)\,q(x)$ (where $\epsilon(x) = \theta(x) - \theta(-x)$). The operator to which the above is then equated is

$$P \int \frac{d\lambda}{i\,\pi}\ \frac{\langle P|\overline{\psi}(0)\sigma^{\mu\nu}\gamma_5\psi(\lambda\,n)|P\rangle}{\lambda}. \tag{8}$$

(b) For an ensemble of *free* quarks (and gluons) the quantity entering in the transverse spin sumrule is related to transverse spin density. One has

$$g_T(x) = \frac{m}{Mx}\,h_1(x). \tag{9}$$

In this way one might establish a relation with transverse spin, but I have to look at this further. It certainly then depends on the assumption that the nucleon can be described as an ensemble of free non-interacting quarks (and gluons), an assumption made in the BLT paper.