Quark Axial Current in Longitudinal and Transverse sum rules

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1 Axial Current and Quark Spin

The appearance of quark axial current in spin sum rules is because it is proportional to quark spin entering quark–gluon angular momentum density

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}. \tag{1}$$

The first term in the r.h.s. is just the canonical quark spin tensor. Note that the energy–momentum tensor (EMT) here accumulates the quark orbital momentum as the *total* gluon angular momentum.

The partonic sum rules emerge because the hadronic matrix elements of conserved quark-gluon operators are fixed. The operator is question is the projection of Pauli-Lubanski vector (PLV)

$$(Sn) \sim \epsilon_{\mu\nu\rho\alpha} n^\alpha \int d^3x M^{\mu,\nu\rho}(x). \tag{2}$$

where n is a vector having only the space components in proton rest frame, so that $nP = 0$. Because the quark spin tensor is totally antisymmetric, the quark spin contribution to PLV is proportional to axial current

$$S^\mu_{quarkspin} = \int d^3x J^\mu_{5S}(x) \tag{3}$$

The spin operator is perfectly local and taking forward matrix element results is the following quark spin contribution to sum rules

$$\Delta q_n =< P, s|S^\mu_{quarkspin}|P, s > n_\mu/ < P, s|P, s >=< p, S|J^\mu_{5S}|P, s > n_\mu/2M \tag{4}$$

Therefore, axial current appears in both longitudinal and transverse sum rules. In the former case vector n should be chosen as a longitudinal one (that is a usual light cone vector in a hard process directed along , say, photon momentum in DIS), resulting in standard longitudinal distribution Δq. The transverse case requires "projecting the Pauli-Lubanski vector on transverse direction" (see [1], before eq. 5) n and results in what is called g_T in the case of structure functions, but, I am afraid does not have commonly accepted name for the case of distributions.

2 Is axial current cancelled from spin sum rules?

The answers is yes and no - this is a typical example of complementarity. We may express the quark spin in the orbital form with the simultaneous change of the energy–momentum tensor discovered by Belinfante long ago

$$M^{\mu,\nu\rho} = x^\nu T_B^{\mu\rho} - x^\rho T_B^{\mu\nu}. \tag{5}$$

Now, the total spin may be expressed in terms of formfactors of symmetric EMT.

$$J_q \sim B_q^S \tag{6}$$

At the same time, the same quantity may be expressed in terms of formfactors of canonical EMT and axial current

$$J_q \sim B_q^C + g_A \tag{7}$$

Of course, if one express B^C in terms of B^S and g_A, and substitute in the last equation, g_A cancels, which is just what is stated in [3] of Elliot's note. This by no means exclude the possibility to use g_A and B^C as independent variables. Experimentally, one may define g_A from inclusive processes and B^S from DVCS etc. So, by combining these two inputs one may determine B_C, that is, quark canonical orbital momentum.

References

[1] O. Teryaev, B. Pire and J. Soffer, arXiv:hep-ph/9806502.