THE BLT TRANSVERSE POLARIZATION ANGULAR MOMENTUM SUM RULE

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1 A little history—the prejudice against a transverse polarization sum rule

In order to construct an angular momentum sum rule one needs an expression, valid in any frame, for the expectation value of the angular momentum operator for a nucleon in a state of definite momentum and spin. In a much cited paper, Jaffe and Manohar [1] stressed the subtleties involved and the need to utilize wave packets to avoid ambiguities. With their result it is clear that no sum rule can exist for the case of transverse polarization.

However, as pointed out by BLT [2], there are errors in the J-M derivation, which Jaffe and Manohar openly and frankly acknowledge. Although the errors are in a sense small technical errors, their consequences are enormous. With the corrected expression there is no difference in the structure of the longitudinal and transverse cases, and very similar sum rules can be derived for both.

The results are:

$$1/2 = 1/2 \sum_{flavours} \int dx \, \{ \, \Delta_T q(x) + \Delta_T \bar{q}(x) \} + \sum_{q,\, \bar{q},\, G} \langle L_{\boldsymbol{s}_T} \rangle \tag{1}$$

which is quite analogous to

$$1/2 = 1/2 \sum_{flavours} \int dx \, \{ \, \Delta q(x) + \Delta \bar{q}(x) \} + \int dx \, \Delta G(x) + \langle L^q \rangle + \langle L^G \rangle \tag{2}$$

Just as one may interpret the $\Delta q(x)$ as the difference between the number density of quarks polarized longitudinally along or opposite to the polarization of a longitudinally polarized nucleon, so the $\Delta_T q(x)$ can be interpreted as the difference between the number density of quarks polarized transversely along or opposite to the polarization of a transversely polarized nucleon.

2 Relation to other processes

The sum rules become particularly interesting if some of the terms on the right hand sides can be measured in hard reactions.Thus the flavour singlet combination of longitudinally polarized quark and antiquark densities in (2) can be extracted from measurements on DIS etc.

However, this combination of longitudinally polarized densities is **NOT** the expectation value of the flavour singlet axial vector current, as is sometimes erroneously stated. The latter current is an anomalous current and its expectation value contains a gluonic component. Indeed, it was shown by Shore and White [3], that the axial current cancels out of the longitudinal angular momentum sum rule. (Although the Shore and White paper, being closely based on Jaffe and Manohar, has errors, its conclusion, as verified in a detailed wave packet treatment by BLT, is correct.)

The flavour singlet combination of transversely polarized parton densities in (1) is also directly related to what can be measured. But, just as for the longitudinal case, it does not correspond to the expectation value of any recognizable local operator.

There *is* a local operator, free of anomalies, which plays a role in the structure of the nucleon, namely

$$\bar{\psi}(0) i\sigma^{\mu\nu}\gamma_5\psi(0) \tag{3}$$

whose expectation value is given by

$$\langle P, S|\, \bar{\psi}(0) i\sigma^{\mu\nu}\gamma_5\psi(0)\, |P, S\rangle = 2\, g_T\, (S^\mu P^\nu - S^\nu P^\mu) \tag{4}$$

where g_T is known as the *tensor charge*, but this is equal to the first moment of the *difference* of quark and antiquark transverse polarized densities i.e.

$$g_T = \int_0^1 dx \sum_{flavours} \{\, \Delta_T\, q(x) - \Delta_T\, \bar{q}(x)\} \tag{5}$$

and thus has nothing directly to do with the expression in the transverse sum rule (1).

3 Conclusion

It should be stressed that, neither in the longitudinal case nor in the transverse case, do the combinations of parton densities that occur in the angular momentum sum rules correspond to the expectation value of one of the local operators that play a role in the structure of the nucleon. Nonetheless the combinations of parton densities, in both cases, do have a suggestive physical interpretation, play a role in various reactions, and thus ought to be measured.

4 Bibliography

[1] R.L.Jaffe and A. Manohar, Nucl. Phys. **B337**, 509 (1990)
[2] B. L. G. Bakker, E. Leader and T. L. Trueman, Phys. Rev. **D70**, 114001 (2004)
[3] G. M. Shore and B. E. White, Nucl. Phys. **B581** 409 (2000). (It was Ben

Bakker who drew my attention to this paper , which was the catalyst for a careful examination of the Jaffe-Manohar paper.)